# BIOMEDICAL SIGNAL ANALYSIS 

A Case-Study Approach

## Updates and Corrections

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## Acknowledgments

I thank my students and readers of my book who indicated errors and parts that were not clear in the book; in particular, I thank Fábio José Ayres, and Alexander Kalinichenko.

Symbols and Abbreviations, p xxix:
Add the following entries:
BP: blood pressure
PVC: premature ventricular contraction

Section 1.2.1 The action potential, pp 5-9: Additional illustrations given in Figures 1 and 2.

Important ions:
$\mathrm{Na}^{+}, \mathrm{K}^{+}, \mathrm{Ca}^{+}, \mathrm{Cl}^{-}$

Body fluids: conductive solutions containing ions

Selective permeability: some ions can move in and out of the cell easily, wheareas others cannot, depending upon the state of the cell and the voltage-gated ion channels.

Figure 1: Selective permeable membrane of an excitable (nerve or muscle) cell.


Figure 2: Resting state and depolarization of a cell.

Section 1.2.3 The Electromyogram (EMG), pp 11 - 14: Additional illustration regarding the motor unit given in Figure 3.

muscle fibers of two motor units

Figure 3: Schematic illustration of two motor units in a muscle.

Section 1.2.3 The Electromyogram (EMG), pp 11 - 12:
In the second paragraph, change the innervation ratio of the platysma from 25 to 15 , and that for the first dorsal interosseus (finger) muscle from 340 to 203. There appears to be some ambiguity in the literature regarding the reference to the motor nerve fiber, the axon of the motor unit, or major nerve fibers serving the motor unit in calculating the innervation ratio.

In the caption of Figure 1.6, add: $\mathrm{ME}=$ Myoelectric signal.

Section 1.2.4 The Electrocardiogram (ECG), p19:
Add an item to the enumerated list on p19 as follows:
4. The AV node fires.

Increment the numbers of the remaining items $4-7$ by one.

Section 1.2.8 The Phonocardiogram (PCG), p37:
In the 12th line from the bottom on p37: change PS to TS.

Section 3.1.1 Random noise, structured noise, and physiological interference, p77-78:

Replace the last sentence on p77 with a new paragraph:
The first-order moment and second-order central moment of the signals $x(t)$ and $y(t)$ are related as

$$
E[y]=\mu_{y}=\mu_{x}+\mu_{\eta}
$$

if $\mu_{\eta}=0$, then $\mu_{y}=\mu_{x}$; and

$$
E\left[\left(y-\mu_{y}\right)^{2}\right]=\sigma_{y}^{2}=\sigma_{x}^{2}+\sigma_{\eta}^{2}
$$

where $\mu$ represents the mean and $\sigma^{2}$ represents the variance of the random process indicated by the subscript.

Correction to the definition of the autocorrelation function on $\mathbf{p 7 8}$, Equation 3.8:

Replace Equation 3.8 and the next line with the following:

$$
\begin{aligned}
\phi_{x x}\left(t_{1}, t_{1}+\tau\right) & =E\left[x\left(t_{1}\right) x\left(t_{1}+\tau\right)\right] \\
& =\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x_{1} x_{2} p_{x_{1}, x_{2}}\left(x_{1}, x_{2}\right) d x_{1} d x_{2}
\end{aligned}
$$

where $x_{1}$ and $x_{2}$ represent the random variables corresponding to the processes $x\left(t_{1}\right)$ and $x\left(t_{1}+\tau\right)$, respectively, and $p_{x_{1}, x_{2}}\left(x_{1}, x_{2}\right)$ is the joint PDF of the two processes. The ACF may be estimated as

## Section 3.1.2 Stationary versus nonstationary processes, p81:

On p81, second sentence in Section 3.1.2:
Replace
"A random process is said to be stationary in the strict sense or strongly stationary if its ensemble averages of all orders are independent of time, that is, they do not vary with time."
with
"A random process is said to be stationary in the strict sense or strongly stationary if its statistics are not affected by a shift in the origin of time."

Section 3.3.1 Synchronized averaging, Illustration of application, p95:

Replace Equation 3.18 with

$$
\gamma_{x y}(k)=\frac{\sum_{n=0}^{N-1}[x(n)-\bar{x}]\left[y(k-N+1+n)-\overline{y_{k}}\right]}{\sqrt{\sum_{n=0}^{N-1}[x(n)-\bar{x}]^{2} \sum_{n=0}^{N-1}\left[y(k-N+1+n)-\overline{y_{k}}\right]^{2}}}
$$

where

$$
\overline{y_{k}}=\frac{1}{N} \sum_{n=0}^{N-1} y(k-N+1+n),
$$

that is, $\overline{y_{k}}$ is the average of the part of the signal $y$ being used in the template matching procedure. The operation above is causal, and valid from $k=N$ to the last sample of the signal $y$.

## Section 3.3.3 Derivative-based operators ..., p115:

In the third line from the top of the text portion on p115, replace Figure 3.26 with Figure 3.27.

Section 3.4.1 Removal of high-frequency noise: Butterworth lowpass filters, pp 120-121:

On p120, in the two lines just above Figure 3.29 , replace " $2 N-1$ multipliers" with " $2 N+1$ multipliers", twice. The same correction should be applied to the captions of Figures 3.29 and 3.30.

Section 3.4.3 Removal of periodic artifacts ..., Equation 3.66 on p135:
change -1.75261 to +1.75261 .

## Section 3.5 OPTIMAL FILTERING: THE WIENER FILTER, p143:

Before Equation 3.95, change "If we now assume that the noise process is statistically independent of the signal process, we have" to "If we now assume that the noise process is statistically independent of the signal process, and that the mean of at least one of the processes is zero, we have"

Section 3.6.2 The least-mean-squares adaptive filter, p150:
In the sentence after Equation 3.116, change "gradient of the noise" to "gradient of the error".

Equation 3.120, p151:
The notation $\bar{x}^{2}(n)\left(\alpha, r(n), \bar{x}^{2}(n-1)\right)$ indicates that the updated value of $\bar{x}^{2}(n)$ is a function of $\alpha, r(n)$, and $\bar{x}^{2}(n-1)$, as defined at the end of the following paragraph.

## Section 3.12. Problem 17 on p173:

Change the impulse response from $\{3,1,-1\}$ to $\{3,1,-1,1\}$. For the sake of periodic or circular convolution, the two signals to be convolved must have the same duration or period (number of samples).

Section 4.3.2 The Pan-Tompkins algorithm for QRS detection, pp 187-188:

Regarding Equations 4.9 to 4.12 , note that the input to the overall highpass filter is $x(n)$, the output of the initial lowpass filter portion is $y(n)$, and the output of the overall highpass filter is $p(n)$. In particular, the transfer function $H_{h p}(z)$ in Equation 4.11 represents the ratio $\frac{P(z)}{X(z)}$. If all of the operations are combined, the input-output relationship of the overall highpass filter is

$$
p(n)=p(n-1)-\frac{1}{32} x(n)+x(n-16)-x(n-17)+\frac{1}{32} x(n-32)
$$

## Section 4.4.1 Detection of EEG rhythms, p194:

Please note the following paragraph on p194:
"When the ACF or the CCF are computed for various shifts, a question arises about the data samples in one of the signal segments beyond the duration of the other. We may add zeros to one of the signals and increase its length by the maximum shift of interest, or we may use the true data samples from the original signal record, if available. The latter method was used wherever possible in the following illustrations."

If the segments of interest are cut out from the full signal and zeros are used in the shifting process to compute the ACF or the CCF, the results will be different from those shown in Figures $4.8-4.12$.

## Section 4.5.1 Coherence analysis of EEG signals, p203:

In Equation 4.27 , replace $X(f) Y^{*}(f)$ with $X^{*}(f) Y(f)$, to be consistent with the definition of $\theta_{x y}(\tau)$ in Equation 4.23.

In the line above Equation 4.28 on p203, insert "magnitude" before "coherence spectrum". Bendat and Piersol [1] use the term "ordinary coherence function" to represent the function on the right-hand side of Equation 4.28 without the square-root operator; Cadzow and Solomon [2] use the term "magnitude squared coherence function" for the same.

Note the following regarding Equation 4.28 defining the coherence spectrum of two signals: If this expression is computed for two individual signals directly, the magnitude of the result will be equal to unity for all $f$, which is incorrect. In order to evaluate the coherence spectrum as defined in Equation 4.28, each spectral density function involved - $S_{x y}, S_{x x}$, and $S_{y y}$ - must be estimated using an averaging procedure applied to several observations of the processes generating the signals [1, 2, 3]. See Section 6.4, pp $287-304$, for procedures for the estimation of spectral density functions.

Section 4.6 The Matched Filter, pp 204-208, Figures 4.15-4.17:
If the derivation of the impulse response of the matched filter is based upon Equation 4.29 and implemented using the DFT, with the number of samples in the DFT equal to the number of samples $N$ in the reference signal or template $x$, due to the periodicity of the DFT, the time delay or shift provided inherently will be $(N-1)$. However, this is short by one sample: the desired shift is $N$ samples to make the reversed signal causal. For this reason, the impulse response of the matched filter illustrated in Figure 4.15 is wrong; the correct impulse response is given in Figure 4.

Due to this change in the impulse response, the outputs of the matched filter shown in Figures 4.16 and 4.17 should be replaced with the correct versions given in Figure 5 and Figure 6, respectively.


Figure 4: To replace Figure 4.15 in the book. Upper trace: The spike-and-wave complex between $0.60 s$ and $0.82 s$ in the c3 channel of the EEG signal shown in Figure 1.23. Lower trace: Impulse response of the matched filter derived from the signal segment in the upper trace. Observe that the latter is a time-reversed version of the former.


Figure 5: To replace Figure 4.16 in the book. Upper trace: The c3 channel of the EEG signal shown in Figure 1.23, used as input to the matched filter in Figure 4.15. Lower trace: Output of the matched filter. See also Figure 4.13.


Figure 6: To replace Figure 4.17 in the book. Upper trace: The $f 3$ channel of the EEG signal shown in Figure 1.23, used as input to the matched filter in Figure 4.15. Lower trace: Output of the matched filter. See also Figure 4.14.

Section 4.8.2 Homomorphic deconvolution, p213:
In the note following Equation 4.38 on p213:
Change $|X(\omega)|$ to $\log [|X(\omega)|]$. It is assumed that the $\log$ is the natural $\log$ (to base e).

Section 5.2.3, p238:
Change "atrial muscle units" to "atrial myocytes."
Change "ventricular muscle units" to "ventricular myocytes" (twice).
Section 5.4.2, p244:
Change "ventricular motor units" to "ventricular myocytes."
Section 5.4.3 ECG waveform analysis, p248:
Please refer to the new illustration in Figure 7 for graphical definitions of the duration, height, offset, and area of the QRS complex.


Figure 7: Graphical definitions of the duration, height, offset, and area of the QRS complex.

Section 5.5.1 Amplitude demodulation, p252:
Equations 5.16 to 5.19 and associated discussion on p252:
change $f_{o}$ to $\omega_{o}$ (13 times).

Section 5.5.2 Synchronized averaging of PCG envelopes, Figure 5.5, p254:

In the caption, please add: $\bar{p} S_{1}=\operatorname{after} \mathrm{S} 1 ; \bar{a} S_{2}=$ before S 2 .

Section 5.8 Application: Analysis of Exercise ECG, p265:
In the third and second lines from the bottom on p265, change "ST level differences of more than $2 m V$ were reported by the program." to "Significant ST level differences were reported by the program. (Elevation or depression of the ST segment by more than 0.1 mV with reference to the baseline is considered to be significant.)"

Section 5.13 Laboratory Exercises and Projects, Project 1, p274:
Change the order of the Butterworth filter from eight to four: MATLAB cannot produce Butterworth lowpass filters as specified in the problem. You may use cutoff frequencies down to about 5 Hz with order $=4$.

Figure 6.8, p298: Replace with the plot in Figure 8.


Figure 8: Log-magnitude frequency response of the Hann2 window illustrated in Figure 6.3. The window width is $N=128$ samples.

## Section 6.5.1 Moments of PSD functions, pp 305-306:

In all definitions of the moments of PSDs in this section,

$$
S_{x x}(k)=\frac{1}{N}|X(k)|^{2},
$$

where $X(k)$ is the DFT of $x(n) . S_{x x}(k)$ represents the PSD of $x(n)$ (according to Equation 6.14). Note the presence of the factor $\frac{1}{N}$ in the definition of the PSD. An additional factor of $\frac{1}{N}$ is required in the definitions of the moments of PSDs in this section, in order to scale the frequency index $k$ to the appropriate frequency in $H z$ in relation to the sampling frequency $f_{s}$.

Two versions of the moments are defined in the equations provided: one using the normalized frequency variable $0 \leq f_{n} \leq 1$ and the other using the DFT index $0 \leq k \leq N-1$. To clarify this, replace the last part of Equation 6.36 with

$$
\int_{f_{n}=0}^{1}\left|X\left(f_{n}\right)\right|^{2} d f_{n}
$$

replace Equation 6.37 with

$$
\overline{f_{n}}=\frac{2}{E_{x}} \int_{f_{n}=0}^{0.5} f_{n} S_{x x}\left(f_{n}\right) d f_{n}
$$

and replace Equation 6.38 with

$$
\overline{f_{H z}}=\left(\frac{f_{s}}{N}\right) \frac{2}{E_{x}} \sum_{k=0}^{N / 2} k S_{x x}(k)
$$

where $\overline{f_{n}}$ and $\overline{f_{H z}}$ indicate the mean frequency in its normalized form and in $H z$, respectively.

Replace the material on p306 up to Equation 6.47 with the following:

- variance $f_{m 2}$ as the second-order moment by using $\left(f_{n}-\overline{f_{n}}\right)^{2}$ in place of $f_{n}$ (the function of frequency that is multiplied with $\left.S_{x x}\left(f_{n}\right)\right)$ in Equation 6.37 or the equivalent expression using $k$ in Equation 6.38; that is

$$
f_{m 2}=\frac{2}{E_{x}} \int_{f_{n}=0}^{0.5}\left(f_{n}-\overline{f_{n}}\right)^{2} S_{x x}\left(f_{n}\right) d f_{n}
$$

or

$$
f_{m 2}=\left(\frac{f_{s}}{N}\right)^{2} \frac{2}{E_{x}} \sum_{k=0}^{N / 2}(k-\bar{k})^{2} S_{x x}(k)
$$

where $\bar{k}=N \overline{f_{H z}} / f_{s}$ is the frequency sample index corresponding to $\overline{f_{H z}}$.

- skewness as

$$
\text { skewness }=\frac{f_{m 3}}{\left(f_{m 2}\right)^{3 / 2}}
$$

where the third-order moment $f_{m 3}$ is computed with $\left(f_{n}-\overline{f_{n}}\right)^{3}$ in place of $f_{n}$ in Equation 6.37, that is,

$$
f_{m 3}=\frac{2}{E_{x}} \int_{f_{n}=0}^{0.5}\left(f_{n}-\overline{f_{n}}\right)^{3} S_{x x}\left(f_{n}\right) d f_{n}
$$

or

$$
f_{m 3}=\left(\frac{f_{s}}{N}\right)^{3} \frac{2}{E_{x}} \sum_{k=0}^{N / 2}(k-\bar{k})^{3} S_{x x}(k)
$$

- kurtosis as

$$
\text { kurtosis }=\frac{f_{m 4}}{\left(f_{m 2}\right)^{2}}
$$

where the fourth-order moment $f_{m 4}$ is computed with $\left(f_{n}-\overline{f_{n}}\right)^{4}$ in place of $f_{n}$ in Equation 6.37, that is,

$$
f_{m 4}=\frac{2}{E_{x}} \int_{f_{n}=0}^{0.5}\left(f_{n}-\overline{f_{n}}\right)^{4} S_{x x}\left(f_{n}\right) d f_{n}
$$

or

$$
f_{m 4}=\left(\frac{f_{s}}{N}\right)^{4} \frac{2}{E_{x}} \sum_{k=0}^{N / 2}(k-\bar{k})^{4} S_{x x}(k)
$$

In order to avoid notational clutter, the same symbols have been used for the two versions each of $f_{m 2}, f_{m 3}$, and $f_{m 4}$; it is understood that similar versions of the moments are used to compute the ratios for skewness and kurtosis.

Section 7.9 Application: Spectral Modeling and Analysis of PCG Signals, p380:

In the fourth line from the bottom, replace "the six categories" with "the six categories considered of normal, AI, MI, MS, ASD, and the combination of AS and AI."

## Caption of Figure 7.36, p382:

Insert the year 1977 before the copyright symbol.

## Caption of Figure 7.37, p384:

Replace "spectral-tracking functions at 100 Hz for S 1 and 150 Hz for S 2 " with "spectral-tracking functions at (a) 100 Hz for S 1 and (b) 150 Hz for S 2 ".

## Caption of Figure 7.38, p385:

Replace "spectral-tracking functions at 100 Hz for S 1 and 150 Hz for S 2 ; normalized spectral-tracking function at 150 Hz for S2" with "spectral-tracking functions at (a) 100 Hz for S 1 and (b) 150 Hz for S 2 ; (c) normalized spectraltracking function at 150 Hz for S 2 ".

Section 8.5.2 ACF distance, p414:
Edit Equation 8.28 as follows:

$$
d_{F}(n)=\frac{\sum_{k=1}^{q}\left|\phi_{T}(n, k)-\phi_{R}(k)\right|}{0.5+\sum_{k=1}^{q} \min \left\{\sqrt{\phi_{T}(n, k)}, \sqrt{\phi_{R}(k)}\right\}}
$$

Section 8.9 Application: Time-varying Analysis of Heart-rate Variability, p441:

According to the standards of measurement, physiological interpretation, and clinical use of heart rate variability by the Task Force of the European Society of Cardiology and the North American Society of Pacing Electrophysiology [4], the frequency bands of HRV are slightly different from those given on p441, as follows:

VLF: $\leq 0.04 H z$;
LF: $0.04-0.15 H z$;
HF: $0.15-0.4 \mathrm{~Hz}$.

Section 9.2.1 Diagnosis of bundle-branch block, p447:
At the end of the note starting on the 16th line of p447, replace "of a QRS wave." by "after the R wave."

The first negative deflection of a QRS complex is called the Q wave; the first (or any) positive deflection of a QRS complex is called the $R$ wave; and any negative deflection after an $R$ wave is called the $S$ wave.

Section 9.4.1 Discriminant and decision functions, p451:
In the line before Equation 9.5, change the final period to a colon.

Section 9.8.1 The leave-one-out method, p464:
In the sentence before Equation 9.63, replace $\mathbf{x}_{k}^{i}$ with $\mathbf{x}_{k}^{(i)}$.

Section 9.9 NEURAL NETWORKS, p465:
In Equation 9.73, on the right-hand side, change $\theta_{j}$ to $\theta_{j}(n)$.

## Bibliography

[1] Bendat JS and Piersol AG. Random Data: Analysis and Measurement Procedures. Wiley, New York, NY, 2nd edition, 1986.
[2] Cadzow JA and Solomon, Jr. OM. Linear modeling and the coherence function. IEEE Transactions on Acoustics, Speech, and Signal Processing, 35(1):19-48, 1987.
[3] Dobie RA and Wilson MJ. Analysis of auditory evoked potentials by magnitude-squared coherence. Ear and Hearing, 10(1):2-13, 1989.
[4] Task Force of the European Society of Cardiology and the North American Society of Pacing Electrophysiology. Heart rate variability: Standards of measurement, physiological interpretation, and clinical use. Circulation, 93(5):1043-1065, 1996.

