

UNIVERSITY OF CALGARY
DEPARTMENT OF ELECTRICAL & COMPUTER ENGINEERING
ENEL 563 BIOMEDICAL SIGNAL ANALYSIS

Mid-Term Examination

Friday, November 7, 2003

ENE 241

Time: 11:00 a.m. – noon

Total: 20 Marks

- NOTE:
1. This is a closed-book exam,
 2. Calculators with text/program storage capabilities are not allowed.
 3. Answer all questions.
 4. In case of problems requiring numerical or algebraic manipulation, show all steps clearly.
 In case of problems requiring descriptive answers, provide clear statements in point form; long essays are not required.
 In case of problems requiring algorithms, provide the reason/logic for each step.
 5. Specify units or dimensions when appropriate.
 6. In drawing plots of signals, spectra, etc. label the axes clearly.

Marks

1. What are the causes and characteristics of ventricular ectopic beats (premature ventricular contractions or PVCs)?

Draw a sample ECG signal including 4 – 5 normal beats and two PVCs.
 Label parts of the signal indicating:

- a) atrial contraction,
- b) ventricular contraction, and
- c) the 'premature' nature of PVCs.

2. The transfer function of a digital filter is given as:

$$H(z) = 1 + 2z^{-1} + z^{-2}.$$

- 1 a) What is the impulse response of the system?
- 2 b) Draw a signal-flow diagram of the system.

Handwritten notes:
 $n = 16 + 9$
 $\delta = 2.3$

- 1 c) Give the input-output relationship in the form of a difference equation.
- 2 d) Derive the frequency response $H(\omega)$ of the system and explain its characteristics.
- 2 ① e) Draw the pole-zero diagram of the system.
- 10 ② f) Compute the output of the system for the input signal with the sample values $\{0, 0, 4, 4, 4, 4, 4, 0, 0\}$ and provide an interpretation of the result.

3. The output $y(n)$ of a digital filter is given as:

$$y(n) = x(n) - x(n-1]$$

where $x(n)$ is the input.

- a) What is the transfer function $H(z)$ of this filter?
- b) What is the gain of the filter at zero frequency (D.C.) and at half of the sampling frequency?

4. A researcher is designing a Wiener filter and is in need of help in obtaining the autocorrelation matrix, defined as:

$$\Phi = E[x x^T].$$

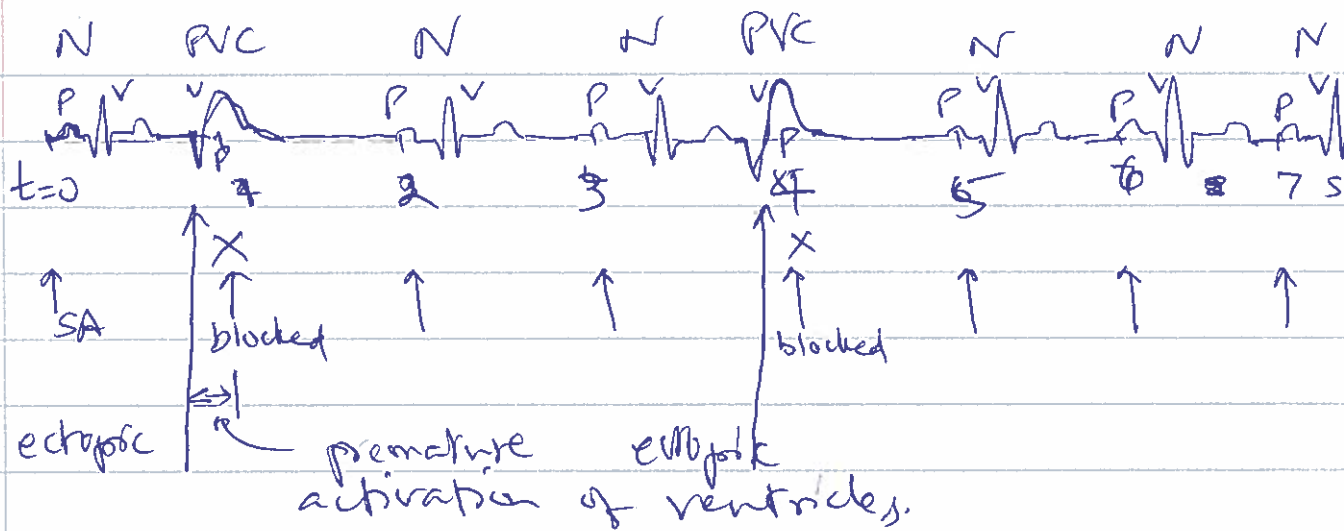
Help the researcher by computing the autocorrelation matrix for the sample vector:

$$x = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix}.$$

- (3) Explain the meaning of the operator $E[\]$ and describe how you would implement the operation in practice.

—xxx—

1. PVCs are caused by abnormal tissue on the ventricles that have gain pacemaker capabilities. Such ectopic foci produce impulses that cause abnormal ventricular contractions. Because PVCs can only occur before a normal pulse of activation arrives from the SA node through the AV node, they are essentially 'premature' contractions. PVCs occur prior to, at the same time, or immediately after atrial contraction due to a normal SA node impulse, but before the SA node impulse is transmitted through the AV node to the ventricles.



N - Normal beat

SA - sino-atrial node pulse

ectopic - ectopic node discharge (pulse)

PVC - premature ventricular contraction

P - atrial contraction

V - ventricular contraction

During a PVC, the ventricular contraction could overlap atrial contraction (occur simultaneously).

Because a PVC occurs before a normal beat would have occurred, it is called 'premature'.

2. $H(z) = 1 + 2z^{-1} + z^{-2}$

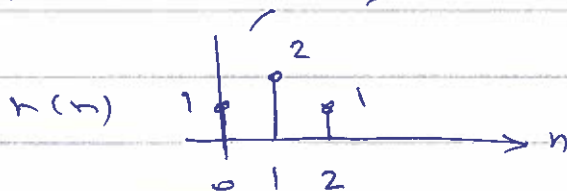
a) impulse response $h(n)$
 $H(z) = ZT[h(n)] = \sum_{n=0}^{\infty} h(n) z^{-n}$

By inspection of $H(z)$, we have

$h(0) = 1, h(1) = 2, h(2) = 1,$

$h(n) = 0, n > 2$

①



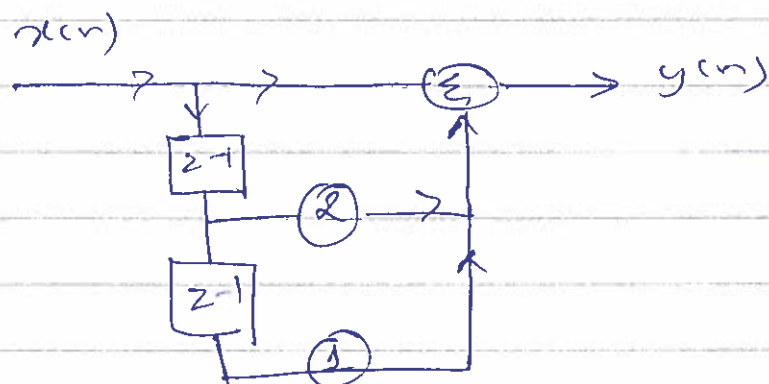
or $h(n) = \delta(n) + 2\delta(n-1) + \delta(n-2)$

b) $H(z) = \frac{Y(z)}{X(z)} = 1 + 2z^{-1} + z^{-2}$

$\therefore Y(z) = X(z) [1 + 2z^{-1} + z^{-2}]$

① c) $y(n] = x(n] + 2x(n-1] + x(n-2]$

②



2. d) $H(\omega) = H(z) \big|_{z=e^{j\omega}}$

$$= 1 + 2\exp(-j\omega) + \exp(-j2\omega)$$

(2) $= e^{-j\omega} [e^{j\omega} + 2 + e^{-j\omega}]$

$$= [2 + 2\cos \omega] e^{-j\omega}$$

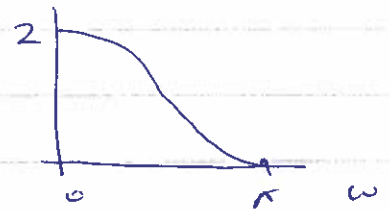
$$|H(\omega)| = |2(1 + \cos \omega)| \quad \text{magnitude}$$

$$\angle H(\omega) = -\omega \quad \text{phase}$$

gain at DC ($\omega=0$) is 4

gain at $\pi/2$ ($\omega=\pi$) is 0

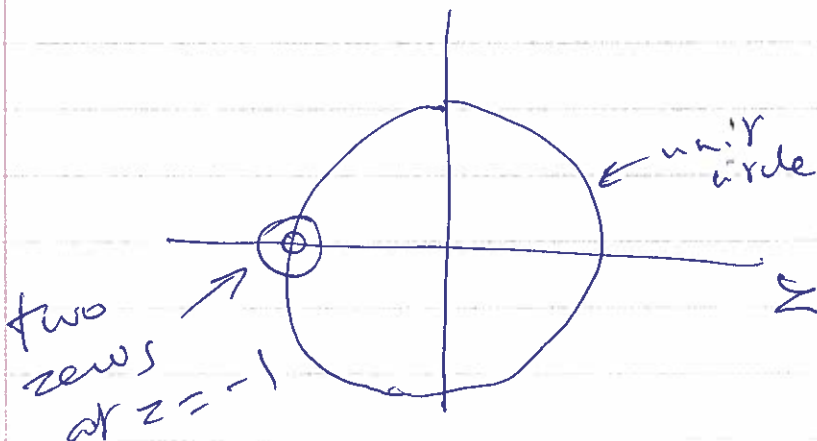
This is a lowpass filter
with linear phase



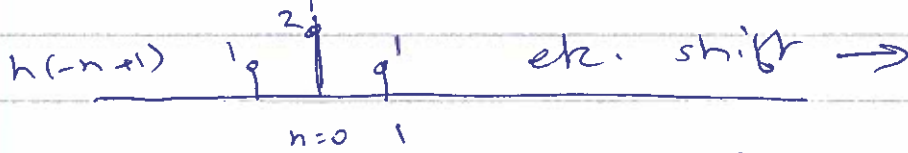
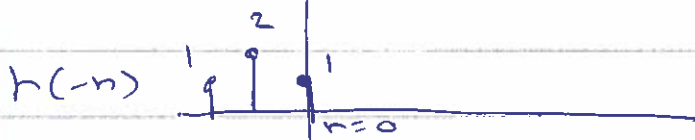
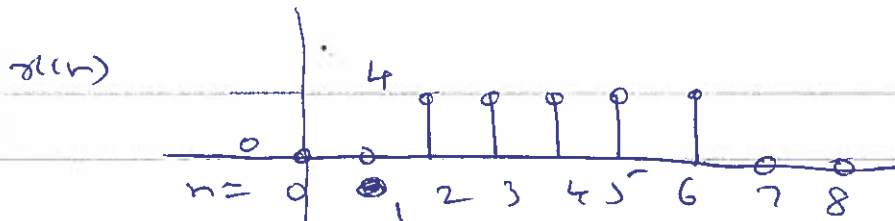
e) $H(z) = 1 + 2z^{-1} + z^{-2}$
 $= z^2 (z^2 + 2z + 1)$
 $= z^2 (z+1)^2$

(2) zeros at $z = -1$ (double zero)

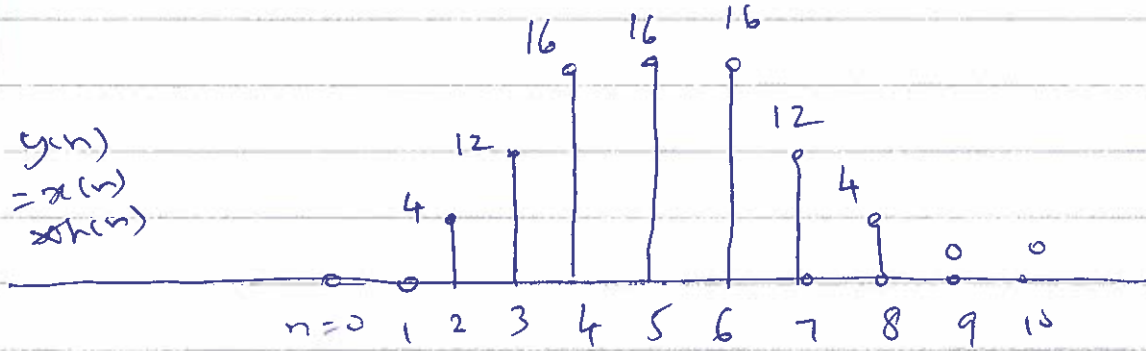
zeros at ∞ and poles at $z=0$ neglected



f)



(2)



The pulse has been smoothed - the upstroke and downstroke are slower in the output.

3 a) $y(n) = x(n) - x(n-1]$

$$Y(z) = X(z) - X(z)z^{-1}$$

$$H(z) = \frac{Y(z)}{X(z)} = 1 - z^{-1}$$

b) DC corresponds to $z = 1$

$$H(z) \big|_{z=1} = 1 - 1 = \underline{\underline{0}}$$

Half of the sampling frequency corresponds to $z = -1$

$$H(z) \big|_{z=-1} = 1 - (-1) = \underline{\underline{2}}$$

$$\underline{x}^T =$$

4. $\underline{x} = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix} \quad [1 \ 2 \ 2 \ 1]$

$$\underline{x} \underline{x}^T = \begin{bmatrix} 1 & 2 & 2 & 1 \\ 2 & 4 & 4 & 2 \\ 2 & 4 & 4 & 2 \\ 1 & 2 & 2 & 1 \end{bmatrix}$$

symmetric

$E[\cdot]$: statistical operation

$$E[y] = \int_{-\infty}^{\infty} y \cdot p(y) dy$$

$$\text{In practice} = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N y_i$$

The ACF matrix may be estimated by computing $\underline{x} \underline{x}^T$ for several signals belonging to the same class (i.e., ECGs of a particular type) and then averaging them.