University of Calgary
Schulich School of Engineering
Department of Electrical and Computer Engineering

ENEL 563 Biomedical Signal Analysis
Midterm Exam

Tuesday, 3 November 2009, ENC 033
9:30 – 10:45 a.m. (75 minutes)
Total Marks: 15

Instructions:

1. This is a closed-book, closed-notes exam.
2. Calculators and electronic devices of any kind are not allowed.
3. Answer all (four) questions.
4. In case of problems requiring numerical or algebraic manipulation, show all steps clearly.
5. In case of problems requiring algorithms, provide the reason or logic for each step.
6. Specify units or dimensions when appropriate.
7. In drawing plots of signals, spectra, etc., label the axes clearly.

Question 1: What is the electroneurogram (ENG)?
Describe an experimental procedure to acquire an ENG.
Describe the typical result obtained from ENGs and a potential application.
(3 marks)

Question 2: A random signal $x(t)$ is characterized by its probability density function (PDF) $p_x(x)$.

Write the basic definition of the mean-squared value of x based upon its PDF.

Given an observation of the signal in terms of its samples, expressed as $x(n)$, $n = 1, 2, \ldots, N$, give a formula to compute the mean-squared value of x (without the PDF).

Give a formula to obtain the root mean-squared (RMS) value of x.

If x represents an electrical signal, what do the mean-squared and RMS values represent?
(3 marks)

Question 3: For a discrete-time signal $x(n)$, write the basic definition of the z-transform.

The signal $x(n)$ is processed by a linear shift-invariant system with the impulse response $h(n)$. Write the complete mathematical expression for the output $y(n)$.

Showing all steps, derive the relationship between the z-transforms of $x(n), h(n)$, and $y(n)$.

(3 marks)

Question 4: A digital filter is specified by the difference equation

$$y(n) = x(n) - x(n-2),$$

where $x(n)$ is the input and $y(n)$ is the output.

Derive the transfer function of the filter.

Derive the magnitude and phase parts of the frequency response of the filter. Let the sampling frequency be normalized to unity.

Plot the magnitude and phase responses. Indicate the values of the functions at normalized frequency values of $0, 0.25$, and 0.5.

Explain the nature and effects of the filter.

(6 marks)

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1. ENG: signal showing propagation of the action potential along a nerve

Procedure:

- apply an electrical stimulus (100 V, 100-300 μs) to a ~~nerve~~ nerve (ex. ulnar nerve)
- record ENGs using surface Ag-AgCl electrodes at ~~three~~ three positions along the nerve (ex. wrist, below elbow, above elbow) (10 Hz - 10 kHz)
- measure delays (latencies) between ENGs
- measure distances between electrodes
- derive propagation velocity

- Result: propagation or nerve conduction velocity. useful in detection of neural diseases

2. MS: $E[x^2] = \int_{-\infty}^{\infty} x^2 \, p_x(x) \, dx$

$$MS = \frac{1}{N} \sum_{n=1}^{N} x^2(n)$$

$$RMS = \sqrt{\frac{1}{N} \sum_{n=1}^{N} x^2(n)}$$

MS: average power

RMS: average magnitude

3. $$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

if causal, with N samples, $X(z) = \sum_{n=0}^{N-1} x(n) z^{-n}$

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k)$$

if causal, $y(n) = \sum_{k=0}^{n} x(k) h(n-k)$

$$ZT[y(n)] = \sum_{n=-\infty}^{\infty} y(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \left[\sum_{k=-\infty}^{\infty} x(k) h(n-k) \right] z^{-n}$$

interchange order of Σ $$= \sum_{k=-\infty}^{\infty} x(k) \sum_{n=-\infty}^{\infty} h(n-k) z^{-n}$$ let $m = n-k$, $n = m+k$

$$= \sum_{k=-\infty}^{\infty} x(k) \sum_{m=-\infty}^{\infty} h(m) z^{-(m+k)}$$

$$= \sum_{k=-\infty}^{\infty} x(k) \left[\sum_{m=-\infty}^{\infty} h(m) z^{-m} \right] z^{-k}$$

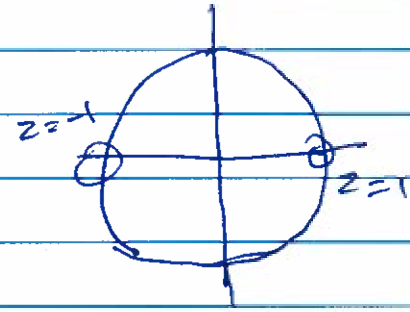
$$= \sum_{k=-\infty}^{\infty} x(k) H(z) z^{-k}$$

$$= H(z) \sum_{k=-\infty}^{\infty} x(k) z^{-k}$$

$$Y(z) = H(z) X(z)$$

4. $y(n) = x(n) - x(n-2)$

$Y(z) = X(z) - z^{-2} X(z)$



① $H(z) = \dfrac{Y(z)}{X(z)} = 1 - z^{-2}$

$H(\omega) = H(z)\big|_{z=e^{j\omega}} = 1 - e^{-j2\omega}$

$= e^{-j\omega}\left(e^{j\omega} - e^{-j\omega}\right)$

$= e^{-j\omega}\, 2j \sin\omega$

$1 - \cos 2\omega + j \sin 2\omega$

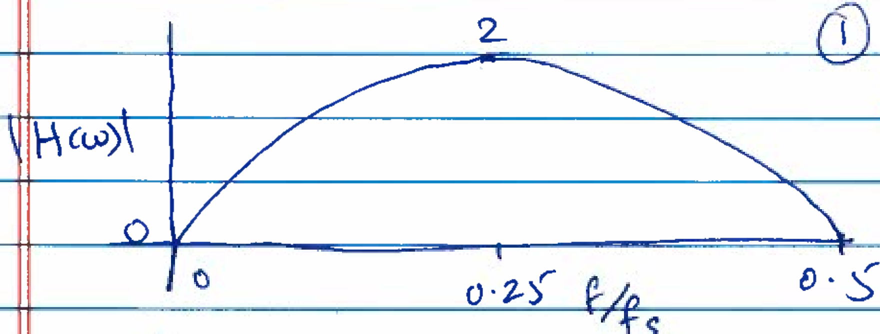
$|H(\omega)|^2 = (1 - \cos 2\omega)^2 + (\sin 2\omega)^2$

$= 1 - 2\cos 2\omega + \cos^2 2\omega + \sin^2 2\omega$

$= 2(1 - \cos 2\omega)$

① $|H(\omega)| = |2\sin(\omega)|$

① $\angle H(\omega) = \frac{\pi}{2} - \omega$

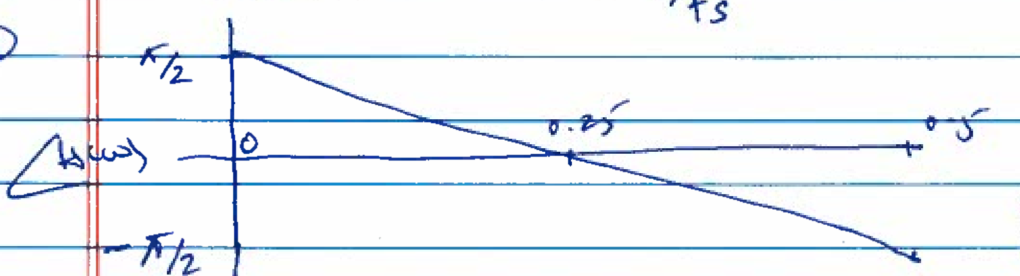
① 

① BPF

HPF parts removes LF artifacts, baseline wander

LPF parts cuts HF noise

Detect QRS in ECG.

① 

on plots

f/f_s	0	0.25	0.5
$\lvert H(\omega)\rvert$	0	2	0
$\angle H(\omega)$	$\pi/2$	0	$-\pi/2$