Assignment-5

Q1

The autocorrelation function (ACF) of a discrete time signal x(n) can be defined as:

$$\Phi_{xx}(m) = \mathcal{E}[\, x(n)\, x(n+m)]$$

$$= \sum_{n=0}^{N-1} x(n)\, x(n+m).$$

To be more precise,

$$\Phi_{xx}(m) = \frac{1}{N - |m|} \sum_{n=0}^{N-|m|-1} x(n)\, x(n+m).$$

m is the delay in samples.

$$\text{PSD} = \mathcal{FT}[\Phi_{xx}(m)] = X^*(\omega)\, X(\omega) = |X(\omega)|^2 = S(\omega)$$

The PSD displays peaks at frequencies corresponding to periodic activities in the signal. This property facilitates the detection of rhythms in EEG signals.

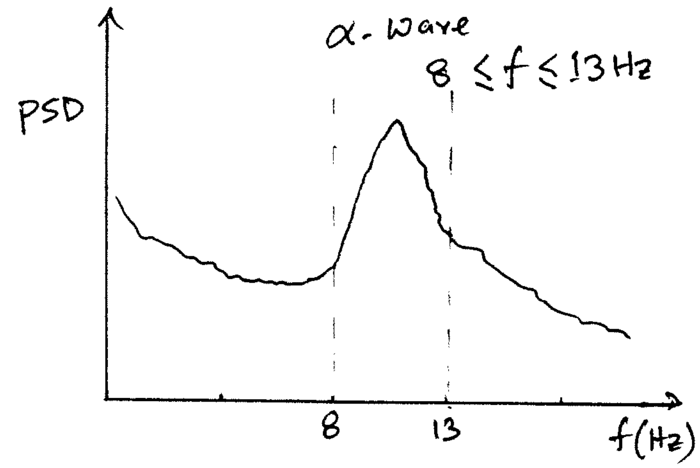
(i) The alpha rhythm should be indicated by a peak or multiple peaks in the neighbourhood of 8 — 13 Hz in the PSD of the EEG signal.

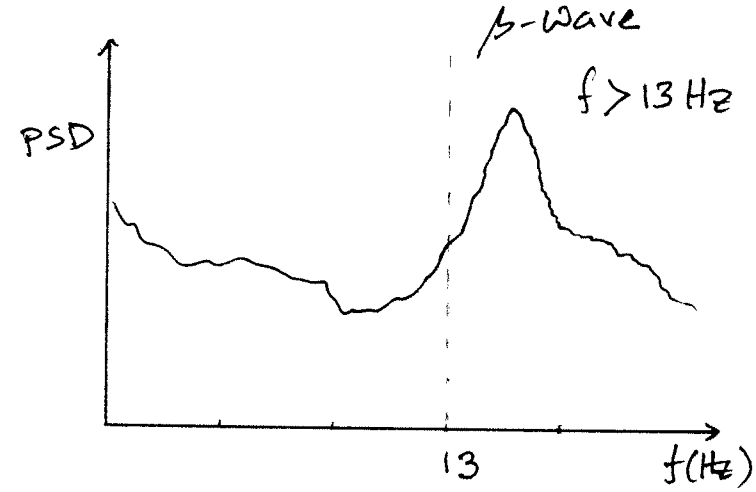
(ii) The beta wave should have a peak or multiple peaks above 13 Hz.

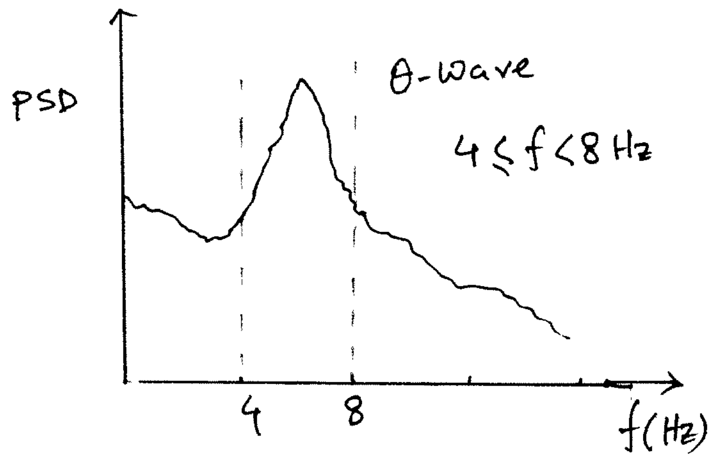
(iii) The PSD of the theta wave should possess a peak or multiple peaks within 4 — 8 Hz.

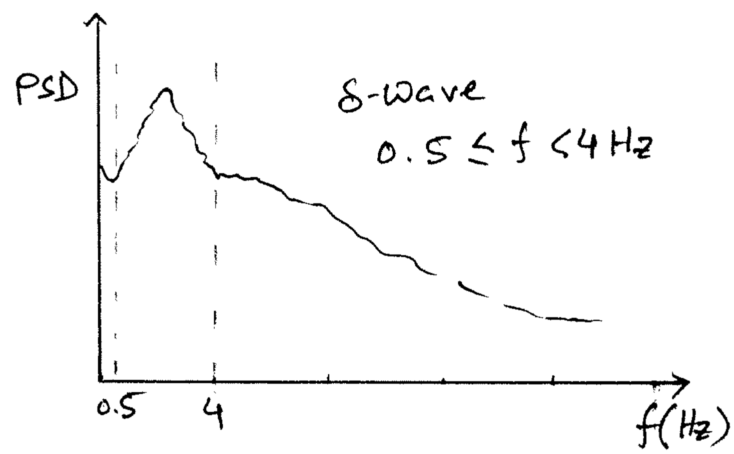
(iv) The delta rhythm should have a peak or multiple peaks in the range 0.5 — 4 Hz.

Schematic representations of PSDs are shown below:









To detect the presence of significant power in a certain frequency band $[f_1, f_2]$, we could compute the fraction of the power of the signal in the specific band, such as

$$E_{f1:f2} = \frac{\int_{f_1}^{f_2} S(\omega)\, d\omega}{\int_{0}^{f_s/2} S(\omega)\, d\omega}$$

If $E_{f1:f2}$ is high, we could say that the signal contains significant power (or EEG rhythm) in the corresponding band.

Q2

A spike and wave complex is a well-defined event in an EEG signal. The complex is composed of a sharp spike followed by a wave with a frequency of about 3 Hz. The wave may contain a half period or a full period of an almost sinusoidal pattern.

We can use template matching method to detect spike and wave complexes in an EEG signal. Template matching can be performed using a normalized correlation coefficient defined as:

$$\gamma_{xy}(k) = \frac{\sum_{n=0}^{N-1} [\,x(n) - \bar{x}\,]\,[\,y(n) - \bar{y}\,]}{\sqrt{\left(\sum_{n=0}^{N-1} [\,x(n) - \bar{x}\,]^2 \sum_{n=0}^{N-1} [\,y(n) - \bar{y}\,]^2 \right)}}$$

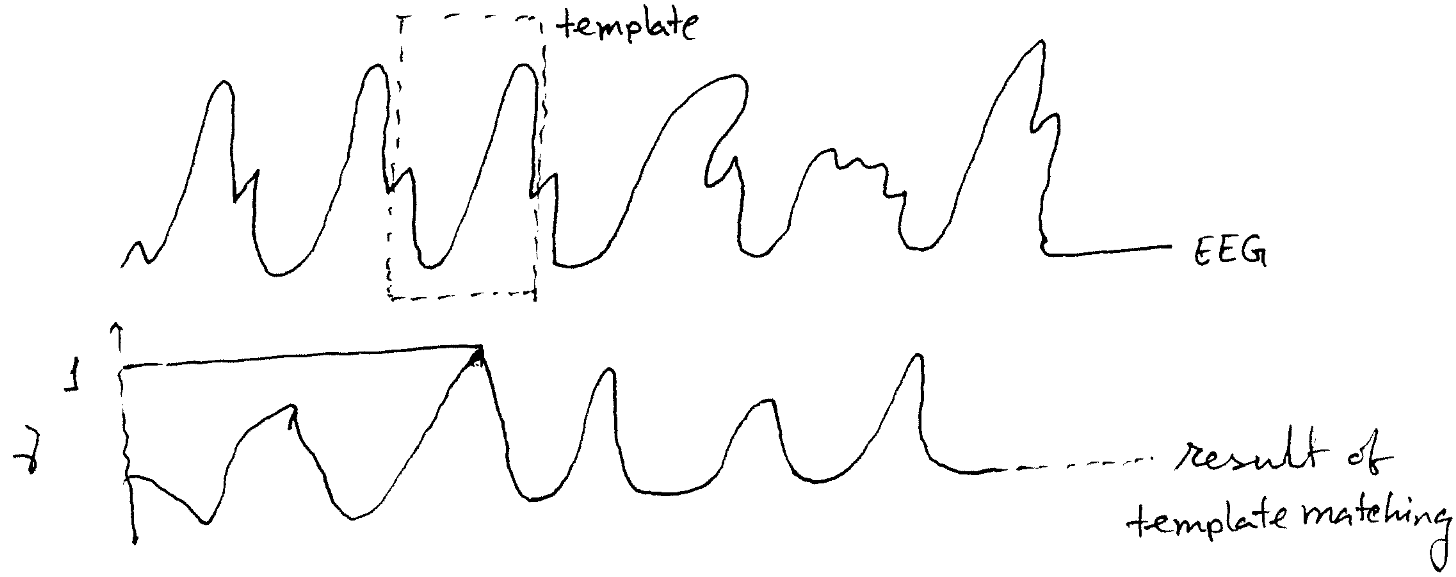
where, x is the template, y is the EEG signal, $\bar{x}$ and $\bar{y}$ are the averages of the corresponding signals over the N samples considered, and k is the time index of the signal y at which the template is placed.

Step-by-step algorithm:-

1. Extract an epoch of a spike and wave complex from an EEG channel.
2. Use the segment for template matching by sliding the template over the entire EEG signal and calculating $\gamma_{xy}(k)$ for each time instants k, using the given formula.
3. A simple threshold on the result would yield the time instants where the events occur.

The template may be correlated with the same channel from which it was extracted to detect similar events that appear at a later time, or with another channel to search for similar events.

The process is illustrated below:



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