

University of Calgary  
Schulich School of Engineering  
Department of Electrical and Computer Engineering  
ENEL 563 Biomedical Signal Analysis  
Final Examination  
Thursday, 19 April 2012  
Noon – 3:00 p.m. (180 minutes)  
Room ENC 127  
Total Marks: 50

**Instructions:**

1. This is a closed-book, closed-notes exam.
2. No calculator or electronic device of any kind is permitted in the exam.
3. Answer all (six) questions.
4. For questions requiring mathematical derivation, show all steps clearly.
5. For questions requiring algorithms, provide the reason or logic for each step.
6. Specify units or dimensions when appropriate.
7. When drawing plots of signals, spectra, etc., label the axes clearly.

**Question 1:** Draw a schematic sketch of a normal electrocardiographic (ECG) signal as well as the corresponding phonocardiographic (PCG) and carotid pulse signals over one cardiac cycle.

Identify the following waves in the signals: P, QRS, T, S1, S2, and the dicrotic notch. Indicate the temporal relationships between the waves mentioned.

Label your figure with the relationships between the waves listed above and the following events in the cardiac cycle: atrial contraction, ventricular contraction, ventricular relaxation, and closure of the aortic valve.

Label the intervals where systolic and diastolic murmurs could appear.

(6 marks)

**Question 2:** Using the continuous-time or discrete-time notation, write the full mathematical expression that defines the output of a linear shift-invariant system in terms of a given input signal and the impulse response of the system.

Starting from the definition of the Fourier transform or the  $z$ -transform, as appropriate, derive the corresponding relationship in the transform domain. Show all steps.

(6 marks)

**Question 3:** A filter is specified in terms of its pole-zero plot as follows: a zero at  $z = 1$  and a zero at  $z = -1$ .

(a) Derive the transfer function of the filter.

(b) Derive the difference equation and draw a signal-flow diagram of the filter.

(c) Derive and plot the impulse response of the filter.

(d) Derive the magnitude and phase of the frequency response of the filter.

(e) Draw a sketch of the magnitude of the frequency response of the filter. Assuming the sampling rate to be 200 Hz, label the frequency axis in Hz.

(f) Derive the gain at 0 Hz and 100 Hz and explain the nature of the filter.

(8 marks)

**Question 4:** A signal of interest,  $x(n)$ , is specified in terms of its samples as  $\{3, 2, 1\}$ , for  $n = 0, 1, 2$ . You are asked to design and demonstrate a matched filter to detect the presence of  $x(n)$  in an arbitrary input signal.

Give the mathematical definition of the impulse response of the matched filter. (A detailed derivation is not required.)

Plot the signal  $x(n)$  and the impulse response of the corresponding matched filter and explain their mutual relationships.

A signal,  $s(n)$ , given by its samples as  $\{0, 0, 0, 0, 6, 4, 2, 0, 0, 0, 0, 3, 2, 1, 0, 0, 0\}$ , for  $n = 0, 1, 2, \dots, 16$ , is processed using the matched filter. Derive the output of the filter, showing all steps.

Plot the signal  $s(n)$  and the output of the filter, and explain the result.

(8 marks)

**Question 5:** Explain the concept of an envelope of a signal.

Give a step-by-step algorithm to compute an envelope of a signal. Include at least two equations for parts of the procedure.

Draw a normal phonocardiographic (PCG) signal over one cardiac signal and a corresponding envelope. Explain the relationships between the two and suggest a use of the envelope in a practical application.

(6 marks)

See the next page for the next question.

**Question 6:** A student new to the area of biomedical signal processing wishes to develop methods to detect the presence of murmurs in phonocardiographic (PCG) signals. The student wishes to derive quantitative measures from segments of the PCG signal to facilitate characterization of murmurs. Help the researcher with the following:

(a) Explain the notion of waveform complexity. Give equations to compute the mean, the variance, and the form factor (FF) of a signal  $x(n)$  with  $N$  samples. Explain how FF may be expected to differ between normal heart sounds and murmurs.

(b) Give a step-by-step algorithm to compute the power spectral density (PSD) of a signal. Include mathematical formulas to compute the discrete Fourier transform (DFT) of the signal and to obtain the PSD from the DFT.

(c) Give typical bandwidths of normal heart sounds and murmurs. Draw schematic diagrams of the PSDs of normal heart sounds and murmurs, and explain the differences between them.

(d) Give a step-by-step procedure to compute the ratio of the power of a given signal in a particular band of frequencies  $[f_1, f_2]$  to the total power of the signal.

(e) Assume that a signal segment has  $N = 500$  samples, that the DFT is computed using the same number of samples, and that the sampling rate is 1000 Hz. For the purpose of detecting murmurs in PCG signals, give a typical range, in Hz, for the band  $[f_1, f_2]$  mentioned above. For the same range of frequencies, give the range of the DFT samples to be used to compute the ratio of power mentioned above. Explain how this measure could be used to detect murmurs and to distinguish murmurs from normal heart sounds.

(16 marks)

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