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Assignment-2

Q1)

Random noise:- Any one of the following:

*Artifact due to the lack of firm contact of the transducer/microphone with the skin (movement and friction).

--May be prevented by attaching the microphone properly.

*High-frequency EM noise picked up from the surroundings due to poor shielding of cables.

-- May be prevented by good shielding of the cables and grounding of the shield.

Physiological artifacts:- Any one of the following:

*Breath sounds- May be prevented if the subject can hold breath during the recording period.

*Bowel sounds due to peristalsis and gas- Difficult to prevent; the patient may be requested to come with an empty stomach.

Q2)

y(t)=ax(t-t1)+s(t)   x(t): signal of interest;
a: scaling factor;
t1: time delay;
s(t): noise or artifact.

Fourier transform:

$$\mathcal{FT}[x(t)] = X(\omega) = \int_{-\infty}^{\infty} x(t)\exp(-j\omega t)\,dt$$

$$\text{So, } Y(\omega)=\mathcal{FT}[y(t)] = \int_{-\infty}^{\infty} y(t)\exp(-j\omega t)\,dt$$

$$= \int_{-\infty}^{\infty} \{ax(t-t_1) + s(t)\}\exp(-j\omega t)\,dt$$

$$= \int_{-\infty}^{\infty} ax(t-t_1)\exp(-j\omega t)\,dt + \int_{-\infty}^{\infty} s(t)\exp(-j\omega t)\,dt$$

For the 1st part,

Let,

$t - t_1 = p \quad \Rightarrow t = p + t_1 \qquad$ if $t = \infty$, $p = \infty$

if $t = -\infty$, $p = -\infty$

$dt = dp$

$$\int ax(t - t_1) \exp(-j\omega t)\, dt = \int_{-\infty}^{\infty} ax(p) \exp\{-j\omega(t_1+p)\}\, dt$$

$$= a \exp(-j\omega t_1) \int_{-\infty}^{\infty} x(p) \exp(-j\omega p)\, dp$$

Here, $\int_{-\infty}^{\infty} x(p) \exp(-j\omega p)\, dp = X(\omega)$

And, $\int_{-\infty}^{\infty} s(t) \exp(-j\omega t)\, dt = S(\omega)$

So, $Y(\omega) = a \exp(-j\omega t_1) X(\omega) + S(\omega)$.

The Fourier transform of y(t) is equal to the sum of the scaled and shifted Fourier transform of x(t) and the Fourier transform of the artifact.

Q3)

The autocorrelation function (ACF) of a signal x(t) can be defined as:

$$\Phi_{xx}(\tau) = \mathcal{E}[x(t)\, x(t + \tau)]$$

Assuming the signal is stationary and ergodic, the time average ACF is defined as:

$$\Phi_{xx}(\tau) = \int_{-\infty}^{\infty} x(t)\, x(t + \tau)\, dt$$

The Fourier transform of a signal x(t):

$$X(\omega) = \mathcal{FT}[x(t)] = \int_{-\infty}^{\infty} x(t) \exp(-j\omega t)\, dt$$

$$\mathcal{FT}[\Phi_{xx}(\tau)] = \int_{\tau=-\infty}^{\infty} \Phi_{xx}(\tau)\exp(-j\omega\tau)\,d\tau$$

$$= \int_{\tau=-\infty}^{\infty} \left[\int_{t=-\infty}^{\infty} x(t)\,x(t+\tau)\,dt\right] \exp(-j\omega\tau)\,d\tau$$

$$= \int_{t=-\infty}^{\infty} x(t) \left[\int_{\tau=-\infty}^{\infty} x(t+\tau)\exp(-j\omega\tau)\,d\tau\right] dt$$

Let,

$t + \tau = p \quad \Rightarrow \tau = p - t \qquad$ if $\tau = \infty$, $p = \infty$

if $\tau = -\infty$, $p = -\infty$

$d\tau = dp$

Then,

$$\mathcal{FT}[\Phi_{xx}(\tau)] = \int_{t=-\infty}^{\infty} x(t) \int_{p=-\infty}^{\infty} x(p)\exp[-j\omega(p-t)]\,dp\,dt$$

Rearranging,

$$= \left[\int_{t=-\infty}^{\infty} x(t)\exp(+j\omega t)\,dt\right] \left[\int_{p=-\infty}^{\infty} x(p)\exp(-j\omega p)\,dp\right]$$

$$= X^*(\omega)\,X(\omega)$$

$$= |X(\omega)|^2$$

= magnitude squared of the frequency response of the signal.

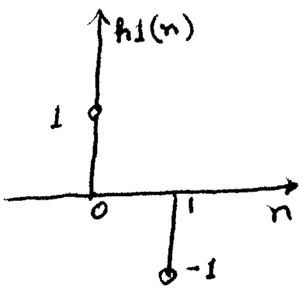
But, a signal's PSD can be defined as, PSD = $|X(\omega)|^2$.

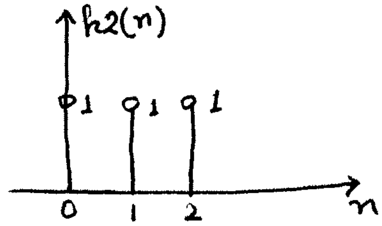
So,

$\mathcal{FT}[\Phi_{xx}(\tau)]$ = PSD of the signal.

Q4)

Let, h1(n)= δ(n) - δ(n-1), and h2(n)= δ(n) + δ(n-1) + δ(n-2).





(a) $H1(z) = \mathcal{Z}\,[\,h1(n)\,] = \sum h1(n)\, z^{-n}$

$= 1 - 1\, z^{-1}$

$= 1 - z^{-1}$

Similarly, $H2(z) = \mathcal{Z}\,[\,h2(n)\,]$

$= 1 + z^{-1} + z^{-2}$

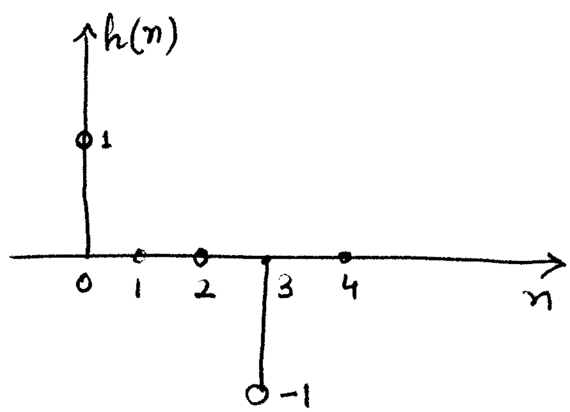
(b) The combined filter transfer function, $H(z) = H1(z)H2(z)$

$= (1 - z^{-1})\,(1 + z^{-1} + z^{-2})$

$= 1 + z^{-1} + z^{-2} - z^{-1} - z^{-2} - z^{-3}$

$= 1 - z^{-3}$

(c) $H(z)= 1 - z^{-3}$



Impulse response of the combined filter,
$h(n) = \mathcal{Z}^{-1}\{H(z)\}$

$= \mathcal{Z}^{-1}\{1\} - \mathcal{Z}^{-1}\{z^{-3}\}$

= δ(n) - δ(n-3).

[Note: You can find the impulse response by using, h(n)= h1(n) ∗ h2(n).]

(d) As both the filters are linear and shift-invariant, the order of the filters will not matter.

In the time domain, $h(n) = h1(n) * h2(n) = h2(n) * h1(n)$.

In the z-domain, $H(z)= H1(z)\, H2(z) = H2(z)\, H1(z)$.

So, the combined filter output will be the same regardless of the order of the filters.

(e)

$$H(z)= \frac{Y(z)}{X(z)}$$

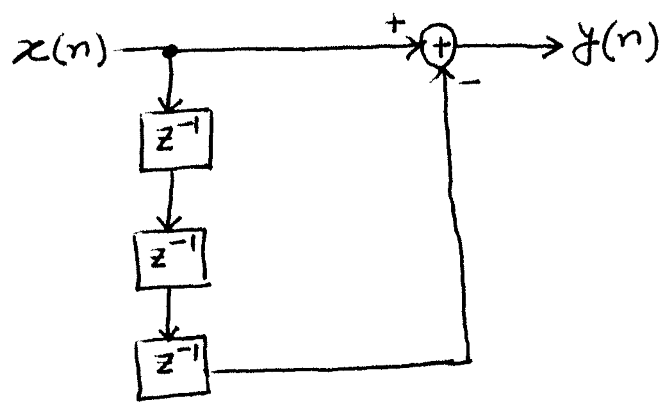
$\Rightarrow Y(z)= H(z)\, X(z)$

$\Rightarrow Y(z)= (1 - z^{-3})\, X(z)$

$\Rightarrow Y(z)= X(z) - z^{-3} X(z)$

Taking inverse z-transform,

$y(n)= x(n) - x(n-3)$.



The signal-flow diagram

(f) $H(z)= 1 - z^{-3}$

$$= \frac{z^3 - 1}{z^3}$$

So, the filter has 3 poles at (0, 0).

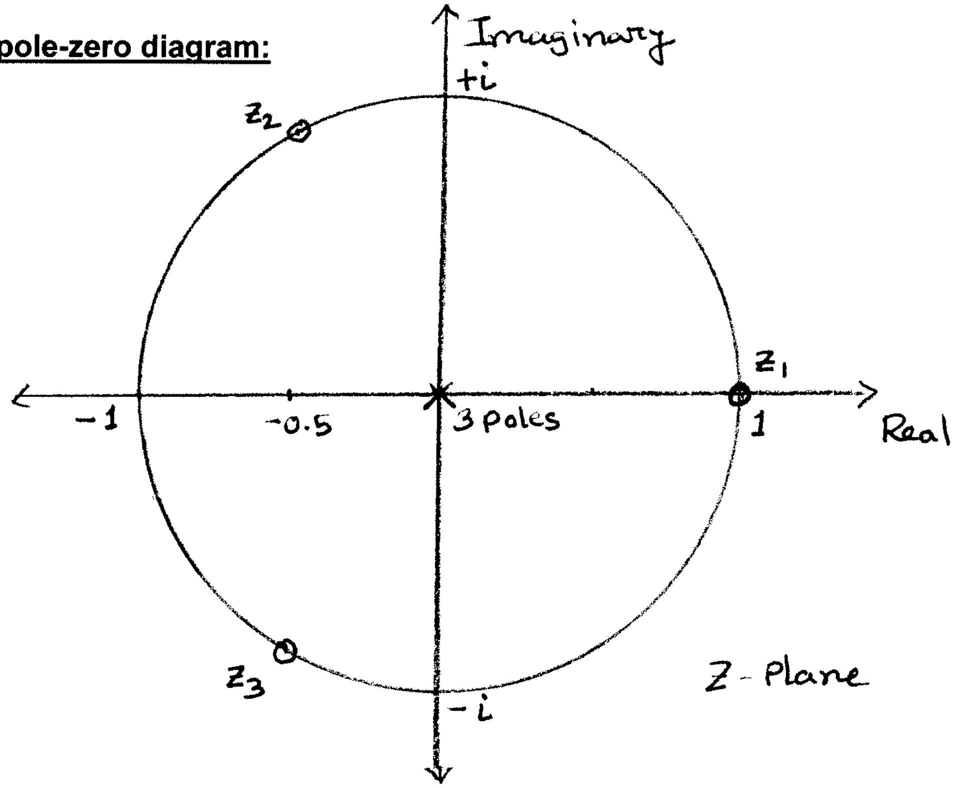
The zeros are,

$z1=1$;

$z2= \frac{1}{2}(-1+\sqrt{3}i) = -0.5+ 0.87i$

$z2= \frac{1}{2}(-1-\sqrt{3}i) = -0.5 - 0.87i$

The pole-zero diagram:



(g) Gain at DC (f=0),

$H(z)|_{z=1} = 1-(1)^{-3}$

$= 0.$

Gain at fs/4,

$H(z)|_{z=i} = 1-(i)^{-3}$

$= 1-(1/i^3)$

$= 1-[1/(-i)]$

$= 1-i.$

$|H(z)|_{z=i}| = \sqrt{(1^2+1^2)}$

$= \sqrt{2}.$

Gain at fs/2,

$$H(z)|_{z=-1} = 1-(-1)^{-3}$$

$$= 1+1$$

$$= 2.$$

The DC gain is zero and the gain is high at fs/2. There is also a notch at ± fs/3. The filter is a combination of a derivative operator, which is a highpass filter, and a moving-average filter which takes the sum of three samples (the same as averaging except for a scale factor), and hence is a lowpass filter. The combined filter, therefore, is a bandpass filter.