RMB

UNIVERSITY OF CALGARY

DEPARTMENT OF ELECTRICAL & COMPUTER ENGINEERING

ENEL 563 BIOMEDICAL SIGNAL ANALYSIS

Final Examination

Wednesday, December 15, 2004 ENA 3 Time: 12:00 – 3:00 p.m.

Total: 50 Marks

NOTE:
1. This is a closed-book exam,
2. Calculators with text/program storage capabilities are not allowed.
3. Answer all questions.
4. In case of problems requiring numerical or algebraic manipulation, show all steps clearly.
   In case of problems requiring descriptive answers, provide clear statements in point form; long essays are not required.
   In case of problems requiring algorithms, provide the reason/logic for each step.
5. Specify units or dimensions when appropriate.
6. In drawing plots of signals, spectra, etc. label the axes clearly.

Marks

3 **1.** Draw a schematic diagram of a motor unit including labels (names) for each part. Explain the generation of the EMG (electromyogram) signal, including the following:

1 a) single motor unit action potential

1 b) innervation ratio

1 c) temporal recruitment, and

1 d) spatial recruitment

(7) Include sketches of at least two sample EMG signals in your discussion.

1 **2.** a) Write an equation to define the cross-correlation between two signals.

2 b) Explain the computational procedures required to obtain the cross-correlation function

4 c) Derive an expression for the Fourier transform of the cross-correlation of two signals in terms of the Fourier transforms of the individual signals. Show all steps. If you use any property of the Fourier transform, give its proof.

(7)

3. The template of a signal is specified by the series of samples { 2, 3, -1 } for n= 0, 1, 2.

3 a) Design a matched filter to detect the signal. Give the impulse response and transfer function of the filter.

3 b) The signal specified by the series of samples { 1, -1, -2, 4, 6, -2, 1, -1, 0 } is applied to the matched filter. Compute the output and explain its characteristics.

(6)

3 **4.** a) Give an equation to define the mean frequency (centroidal frequency) of a power spectral density (PSD) function.

Explain the role of each item of your equation.

2 b) Explain how you would implement the procedure to obtain the mean frequency of a signal.

2 c) Draw schematic sketches of two PSD functions and indicate their approximate mean frequencies. Explain the difference between the two examples.

(7)

5. Describe the significance of the P wave in the analysis of ECG (electrocardiogram) signals.

Describe a method for the detection of P waves in an ECG signal. Explain the purpose and reasoning behind each step of your algorithm. Give at least one nontrivial equation representing a procedure in your algorithm.

(8) Draw schematic sketches representing a sample input signal and the corresponding output at each stage of your method.

6. Propose an algorithm to perform the segmentation of PCG (phonocardiogram or heart sound) signals into four parts per cardiac cycle as:

a) the first heart sound (S1)
b) systolic murmur (SM), if present
c) the second heart sound (S2), and
d) diastolic murmur (DM), if present

If your proposal includes the use of other signals, explain the need and rationale for the use of such signals. Explain the relationship between events in your reference signals and the events of interest in the PCG signal. Provide sketches of typical signals and the results of your methods to illustrate your procedures.

(15) Document your procedures using a flowchart or an algorithmic listing. Give at least three nontrivial equations representing important steps in your procedures.

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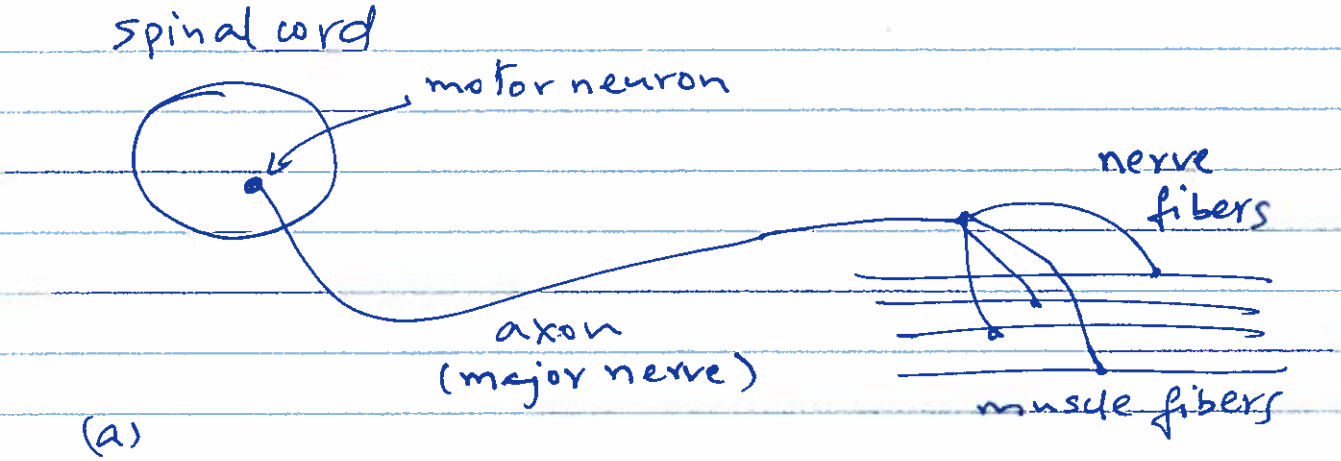
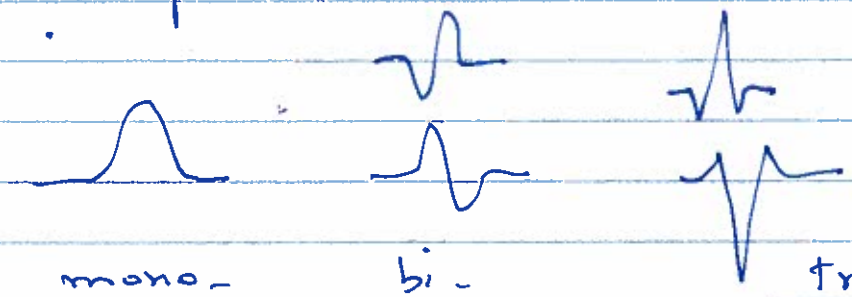
1. Motor unit (MU)



(a)

The EMG is generated when a motor neuron sends a stimulus through the axon to the muscle fibers innervated by the motor neurons. The fibers are triggered or fired into contraction, thereby generating a single motor unit action potential (SMUAP).

SMUAPs may be monophasic, biphasic, or triphasic.



Duration: 3-15 ms; amplitude 100-300 μV

firing rate 6-30 per second.

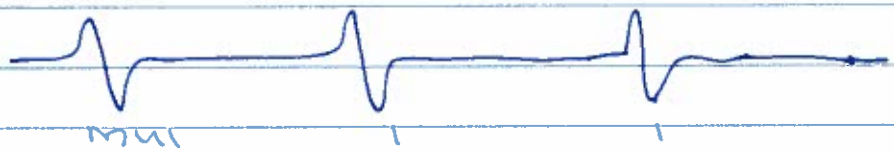
Abnormal SMUAPs may be polyphasic or splintered.

(b) innervation ratio : The number of muscle fibers per ~~motor~~ major nerve fiber.

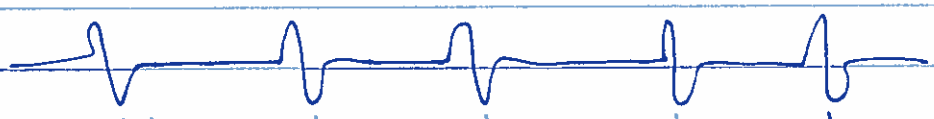
(c) Temporal recruitment: The firing rate of the MUs already activated is increased to obtain increased muscle force

(d) Spatial recruitment: More MUs are activated to generate more muscular force or output.

EMG 1:

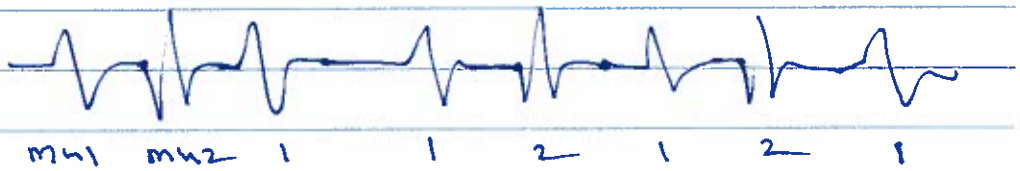


EMG 2:
Temporal recruitment



firing rate increased

EMG 3:
spatial recruitment



MU2 starts firing

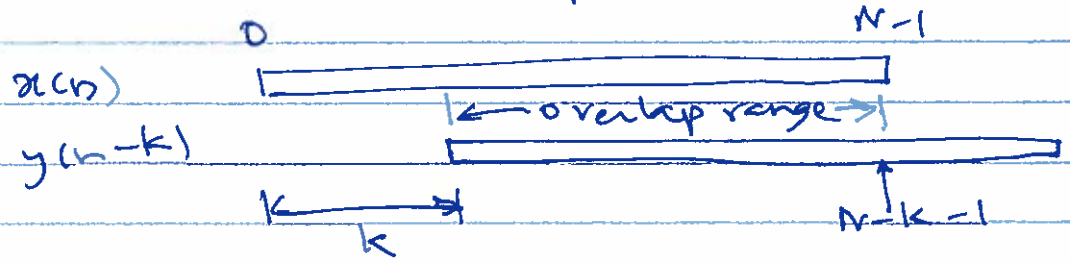
2. a) $\theta_{xy}(\tau) = \int_{-\infty}^{\infty} x(t)\, y(t+\tau)\, dt$

or $\theta_{xy}(k) = \sum_{n=0}^{N-k+1} x(n)\, y(n+k)$

where N is the number of samples available in each signal (0, 1, 2, ..., N-1).

b) CCF involves the following operations:
- shift one of the signals by k samples
- multiply the two signals point by point
- add the products
- repeat for all shifts as required

care must be taken to limit the multiplication and summation to the range of signal data available (that overlap).



2(c) $S_{xy}(\omega) = \int_{\tau=-\infty}^{\infty} \theta_{xy}(\tau) \exp(-j\omega\tau)\, d\tau$

$$= \int_{\tau=-\infty}^{\infty} \left[\int_{t=-\infty}^{\infty} x(t)\, y(t+\tau)\, dt \right] \exp(-j\omega\tau)\, d\tau$$

$$= \int_{t=-\infty}^{\infty} x(t) \int_{\tau=-\infty}^{\infty} y(t+\tau) \exp(-j\omega\tau)\, d\tau\, dt$$

(interchanging order of $\int$ w.r.t. t and τ)

Now, in the integral w.r.t. τ as marked above, change variables as $t+\tau \to \alpha$. Then $d\tau = d\alpha$; $\tau = \alpha - t$

$$\int_{\tau=-\infty}^{\infty} y(t+\tau) \exp(-j\omega\tau)\, d\tau$$

$$= \int_{\alpha=-\infty}^{\infty} y(\alpha) \exp[-j\omega(\alpha-t)]\, d\alpha$$

$$= \int_{\alpha=-\infty}^{\infty} y(\alpha) \exp(-j\omega\alpha)\, d\alpha \cdot \exp(j\omega t)$$

$$= Y(\omega) \exp(j\omega t) \quad \text{where } Y(\omega) = FT[y(t)]$$

Then, $S_{xy}(\omega) = \int_{t=-\infty}^{\infty} x(t) \exp(j\omega t)\, dt\; Y(\omega)$

The integral above is the same as the definition of the FT, but with $\exp(-j\omega t)$ changed to $\exp(+j\omega t)$, that is, $\exp^*(-j\omega t)$. Assuming $x(t)$ to be real, this is $X^*(\omega)$.

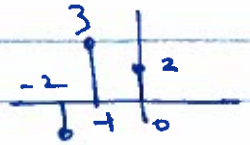
Therefore $FT[CCF] = X^*(\omega)\, Y(\omega)$

3. a) matched filter to detect $x(n)$ is given by $h(n) = \alpha\, x(n_0 - n)$.

$x(n) = \{2, 3, -1\}$

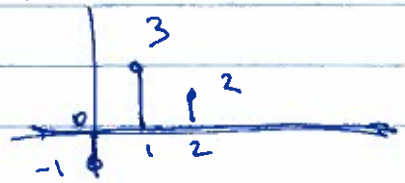
Let $\alpha = 1$

$x(-n)$:



shift by two samples ($n_0 = 2$) to the right:

$h(n) =$



$\{-1, 3, 2\}$ or $-1.\delta(n) + 3\delta(n-1) + 2\delta(n-2)$

$n = 0, 1, 2$

Transfer function $H(z) = \sum_{n=0}^{2} h(n) z^{-n}$

$H(z) = -1 + 3z^{-1} + 2z^{-2}$

b) input $s(n) = \{1, -1, -2, \boxed{4, 6, -2}, 1, -1, 0\}$

$n = 0\ 1\ 2\ \boxed{3\ 4\ 5}\ 6\ 7\ 8$

convolve: $2\ 3\ -1 \rightarrow$ shift

max

output: $-1\ \ 4\ \ 1\ -12\ \ 2\ \ (28)\ \ 5\ \ 0\ \ -1$

$n = 0\ \ 1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6\ \ 7\ \ 8$

$\{4, 6, -2\} \rightarrow 2 \times [2, 3, -1]$

max output at $n = 5$ corresponds to the detection of the pattern $2x(n)$ that ends at $n = 5$ in the input.

Ex: a) mean freq

$$\bar{f} = f_s \frac{2}{E_x} \int_{f=0}^{0.5} f\, S_{xx}(f)\, df \quad -(1)$$

$$\bar{f} = f_s \frac{2}{N E_x} \sum_{k=0}^{N/2} k\, S_{xx}(k) \quad -(2)$$

$$E_x = \sum_{n=0}^{N-1} |x(n)|^2 = \sum_{k=0}^{N-1} S_{xx}(k)$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} |X(k)|^2$$

total energy of signal

The PSD is normalized by dividing by E_x, which is the area under the curve, such that the normalized PSD has an area of unity, and can be treated as a PDF. Then, the mean frequency is given by its first moment: multiply by the frequency f and integrate. However, due to ~~the~~ symmetry of the PSD over +ve and -ve frequencies, the integration is done over one half of the PSD only: 0 to $f_s/2$ or 0 to 0.5 in normalized frequency. For this reason, the energy normalization is done with $E_x/2$. If the frequency has been normalized the result should be scaled by f_s, as in Eqn. (1).

b) 1 - Take FFT of the given signal. Length = N

2 - Take magnitude squared of each element or component. This is the PSD. (and ÷ N)

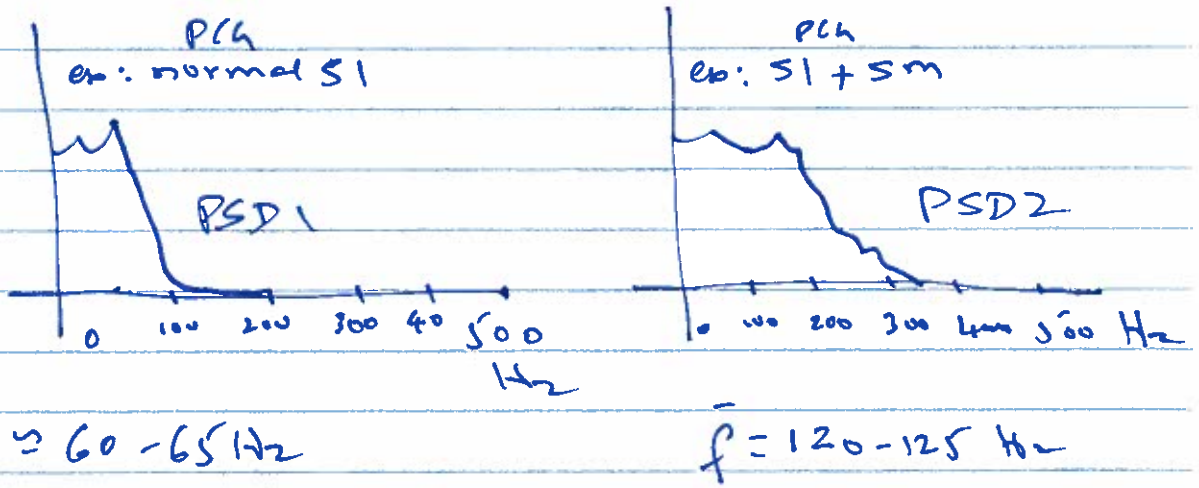
3 - Compute E_x as the sum of all of the elements of the PSD

4 - From k = 1 to N/2, multiply each element of the PSD by k and sum, as in Eqn (2)

5 - multiply the result by $\frac{2f_s}{NE_x}$ to get the mean freq.

If several observations of the signal are available, average the PSD, of the signals before step 3. It is also advisable to remove the DC of the signal.

c)



$\bar{f} \simeq 60 - 65$ Hz

$\bar{f} = 120 - 125$ Hz

PSD2 has more high-frequency energy than PSD1, and hence has a higher mean frequency.

5. P wave represents atrial contraction. Analysis of the P wave and the P-Q interval could assist in the detection of atrio-ventricular conduction problems (desynchronization or dissociation). Atrial flutter and AV dissociation cannot be detected by analyzing only the QRS complex.

method to detect P waves.

1. Using Pan-Tompkins method, detect QRS complexes. Delete them and replace with baseline (PQ segment)

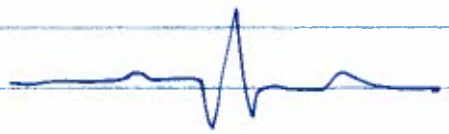
2. BPF 3-11 Hz

3. Search interval: from end of previous T wave to onset of current QRS

4. Rectify and threshold signal in search interval at 50% and 75% of max. to make a ternary signal

5. CCF with ternary template of a representative P wave

6. Detect peak in CCF → P wave location.



ECG

QRS detected, deleted

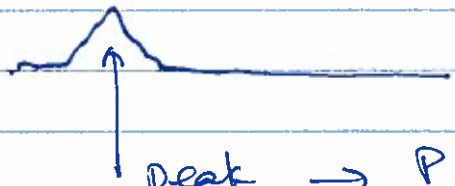
BPF + amplified

2

rectified

Ternary

Ternary Template of P wave



CCF

peak → P wave located

Equations:

1. CCF: $\theta(k) = \sum_{n} t(n+k)\, s(n)$

t: template, s: signal, k: shift or delay
Σ: over the range of overlap between t & s.

2. BPF: LPF and HPF

LPF: $y(n) = \frac{1}{m} \sum_{k=0}^{m-1} x(n-k)$

m: number of samples to be averaged (window width).

HPF = 1 − LPF

6. To perform the segmentation of PCG signals into four parts as required, we could consider the following general steps:

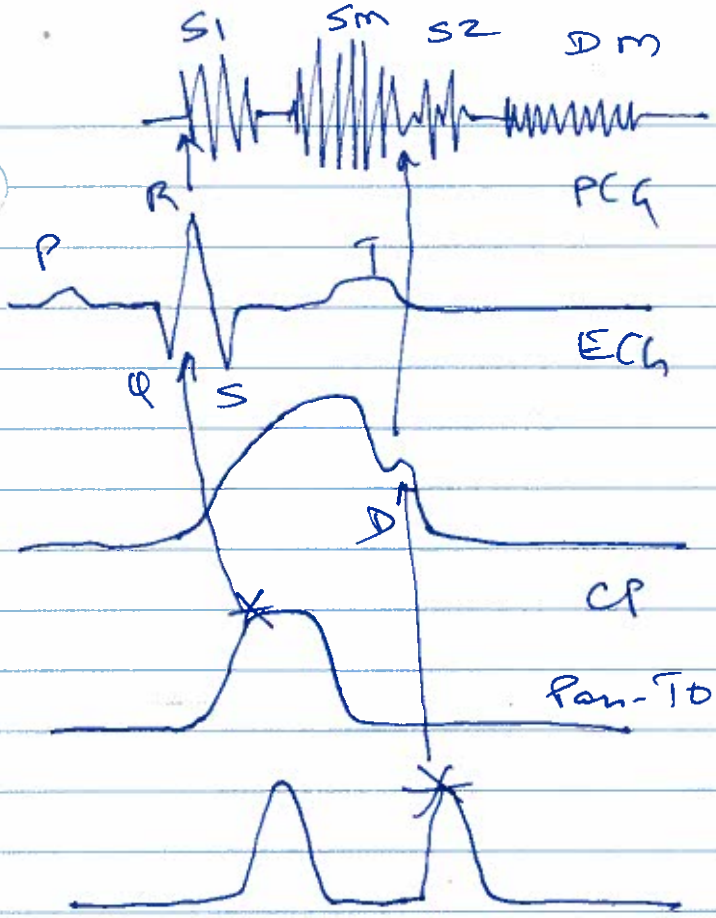
a) Detect S1 by using the ECG. The QRS complex indicates the beginning of ventricular contraction, which is also when S1 begins. The QRS complex may be detected using the Pan-Tompkins method, described below.

b) S1 is about 100-120 ms in duration. Systole is about 350 ms in duration. The beginning of S2 may be detected using the carotid pulse (CP) for reference. The dicrotic notch in the CP relates to the closure of the aortic valve, which is also the beginning of S2 (unless there is A2-P2 reversal). The dicrotic notch may be detected using the smoothed, squared, second derivative of the CP - described below.

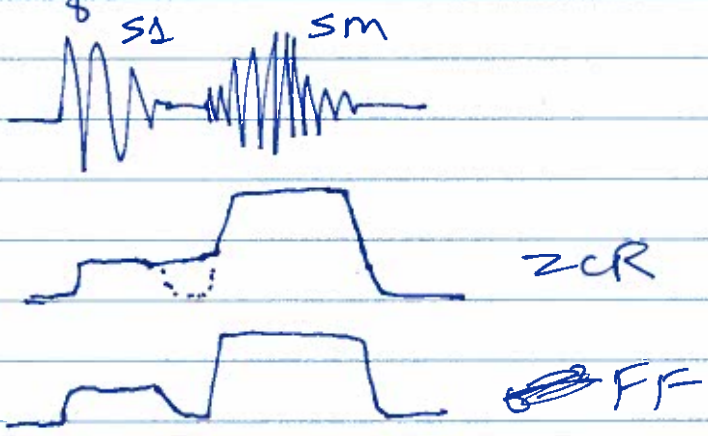
b & d) Once S1 & S2 are detected as above, we have broken the PCG into the systolic and diastolic parts. The systolic part contains S1 + SM (if present)

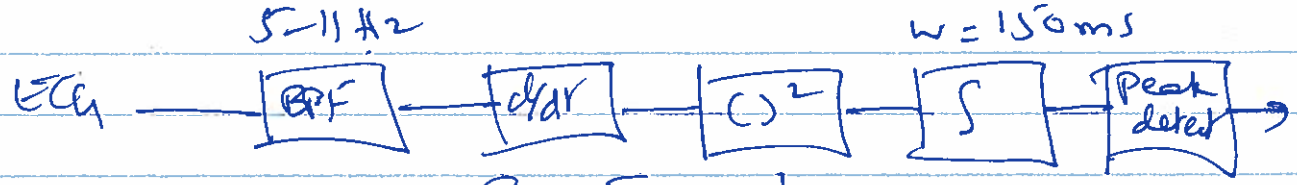
The diastolic part contains S2 + DM if present. murmurs are of higher frequency content than S1 or S2. Therefore, we could use a measure of activity such as zero crossing rate (ZCR) or ~~form factor~~ variance (FF) in a moving window of about 10 or 20 ms width. An increase in ZCR or FF from the initial values will indicate the presence of a murmur. For this purpose, we could use the first 80 - 100 ms of the systolic or diastolic parts for reference. The point where FF or ZCR increases by at least 25% with reference to its average over the S1 or S2 portions could be taken to be the beginning of SM or DM. If the ZCR or FF value does not change much, then we could say that there is no murmur.

Note that the mean of the PCG segment must be subtracted before computing the ZCR.

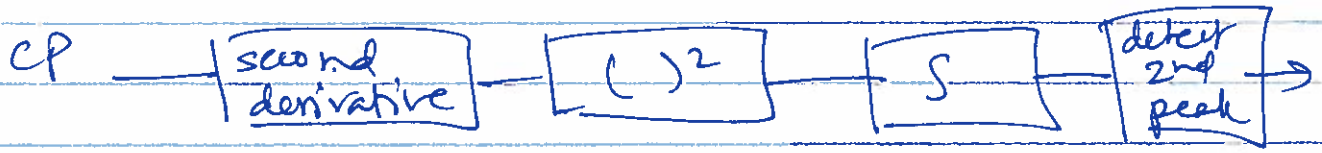


Eg. of murmur detection.

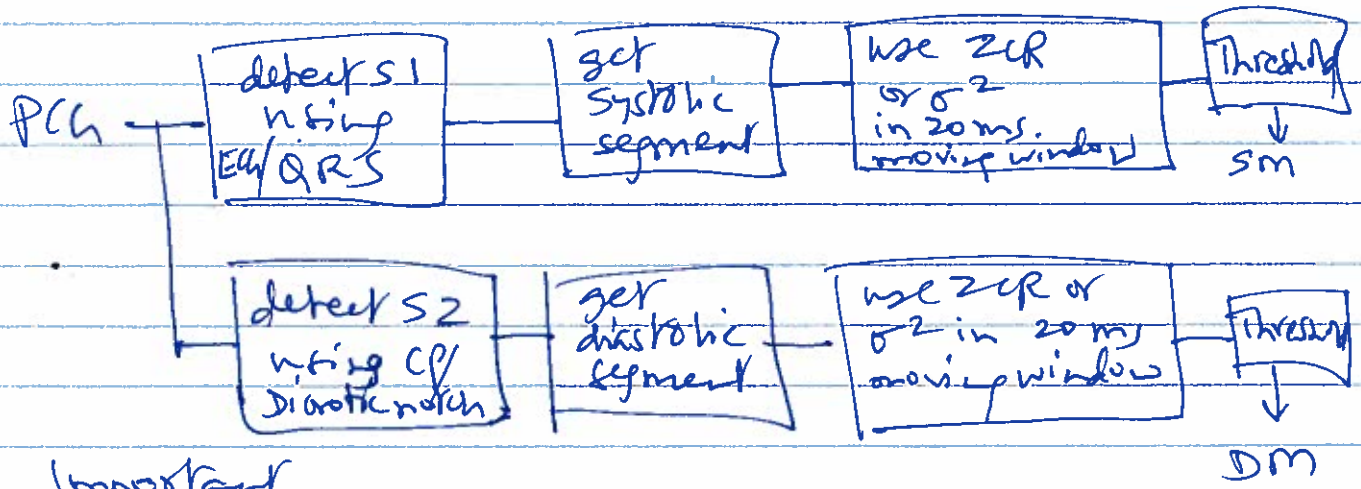




Pan Tompkins method to detect QRS



apply delay corrections from D to S2.



Important Equations

Pan Tompkins : LPF. $H(z) = \frac{1}{32} \frac{(1-z^{-6})^2}{(1-z^{-1})^2}$

$f_s = 200$ Hz

HPF : $H(z) = z^{-16} - \frac{1}{32} \frac{(1-z^{-32})}{(1-z^{-1})}$

Derivative: $y(n) = \frac{1}{8}\left[2x(n) + x(n-1) - x(n-3) - 2x(n-4)\right]$

Control Pulse - detect Dicrotic Notch

second derivative

$$p(n) = 2y(n-2) - y(n-1) - 2y(n) - y(n+1) + 2y(n+2)$$

$$s(n) = \sum_{k=1}^{M} p^2(n-k+1)\, w(k)$$

$$w(k) = (M-k+1), \quad k = 1, 2, \ldots, 16$$

for $f_s = 256$ Hz

$$FF = \frac{\sigma_{x''} / \sigma_{x'}}{\sigma_{x'} / \sigma_{x}}$$

σ^2: variance ; x: signal in window

x': 1st derivative ; x'': second derivative