ENEL563
Assignment 4 Solution

Ans. Q1(a)

$$W(\omega) = \frac{S_d(\omega)}{S_d(\omega) + S_\eta(\omega)} = \frac{1}{1 + \frac{S_\eta(\omega)}{S_d(\omega)}}; \tag{1}$$

Here,

$W(\omega)$: frequency response of the Wiener filter,

$S_d(\omega)$: PSD of the desired (original) signal,

$S_\eta(\omega)$: PSD of the noise,

$\frac{S_d(\omega)}{S_\eta(\omega)}$: SNR as a function of frequency, ω.

Ans. Q1(b)

To derive the information required to design a Wiener filter, we need to estimate the temporal or spectral statistics or model the ACF or the PSD of the signal and the noise.

To acquire the required statistical information about the signals, we can do any one of the following:

I. Find noise-free signals from the database and calculate the ACF or PSD of the signals. Then, average the PSDs and fit a suitable model for the PSD of the signal.

II. If no noise-free signal is available, we can estimate the averaged ACF of several noisy ECG signals. We could perform smoothing of the ACF, and considering the decaying nature of the ACF we could fit a model (e.g., a Laplacian). Then, taking the Fourier transform of the ACF model will give us an estimate of the PSD.

III. If any of the above is not applicable, we can take a segment of the ECG signal or sketch an ECG signal with all the important waves and characteristics. Then, we could use the ACF or PSD of the ECG signal.

To acquire the required statistical information about the noise, we can do any one of the following:

I. We can take portions from the noisy ECG signals where there is no signal content present, such as the flat segments from the end of each T wave to the beginning of the following P wave. Then, we can find the ACF or PSD of the noise and average them for a number of segments. This will give us an approximation of the statistics regarding the noise process.

II. If possible, we can estimate the noise statistics (e.g., variance) from the recording equipment with no patient connected to it. Then, we can fit a model for the PSD of the noise process.

Ans. Q1(c)

From Equation 1 it is obvious that the Wiener filter's gain is inversely related to the noise PSD and directly related to the signal PSD. The filter works adaptively according to the SNR at a particular frequency.

i) When the signal component at a particular frequency is zero, $S_d(\omega) = 0$

$W(\omega) = \frac{0}{0+S_\eta(\omega)} = 0$

So, the gain becomes zero at that frequency.

ii) When the noise component at a particular frequency is zero, $S_\eta(\omega) = 0$

$W(\omega) = \frac{S_d(\omega)}{S_d(\omega)+0} = 1$

So, the gain becomes high (unity) at that frequency.

iii) When the noise component at a particular frequency is much stronger than the corresponding signal component, i.e., $S_\eta(\omega) \gg S_d(\omega)$, then the SNR becomes very low, and

$$W(\omega) = \frac{1}{1 + \frac{S_\eta(\omega)}{S_d(\omega)}} = \frac{1}{1 + a\ large\ value} \approx 0.$$

So, the gain becomes very low for that frequency.

Ans. Q2

Minimum output power of the adaptive noise canceler:

$$min\ E[e^2(n)] = E[v^2(n)] + min\ E[\{m(n) - y(n)\}^2], \tag{2}$$

(a) Minimizing $E[e^2(n)]$ corresponds to minimizing $E[\{m(n) - y(n)\}^2]$, as the signal power $E[v^2(n)]$ is fixed for a given input. So, minimizing the total output power minimizes the output noise power. Note that $m(n) - y(n) = e(n) - v(n)$; both sides of this expression indicate the noise in the output.

(b) Because, $e(n) - v(n) = m(n) - y(n)$, when $E[\{m(n) - y(n)\}^2]$ is minimized, $E[\{e(n) - v(n)\}^2]$ is also minimized. So, adjusting the filter to minimize the total output power is equivalent to causing the output $e(n)$ to be the MMSE estimate of the signal of interest $v(n)$ for the given structure and adjustability of the adaptive filter and for the given reference input.

As a result, the output $e(n)$ will contain the signal of interest $v(n)$ and some noise, i.e., $e(n) = \tilde{v}(n)$. From Equation 2, the output power is minimum when $E[e^2(n)] = E[v^2(n)]$, and at this condition $E[\{m(n)-y(n)\}^2] = 0$. Then, $y(n) = m(n)$ and $e(n) = v(n)$; the output is a noise-free estimate of the desired signal.

(c) From part (a), minimizing the total output power minimizes the output noise power. As the signal power $E[v^2(n)]$ remains unaffected, minimizing the total output power maximizes the output SNR.

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